Unit 2: Kinematics

Problems

2.1 A train moves with a uniform velocity of 36 kmh^{-1} for 10s. Find the distance traveled by it.

Given Data

Velocity of train = V = 36 kmh^{-1} = $\frac{36 \times 1000}{3600}$ = 10 ms^{-1}

Time taken = t = 10 s

Required

Distance travelled by train = S = ?

Solution

As we know that

$S = V \times t$

By putting the values, we have

$S = 10 \times 10$

$S = 100$ m

Result

Distance travelled by train = S = 100 m

2.2 A train starts from rest. It moves through 1 km in 100s with uniform acceleration. What will be its speed at the end of 100s.

Given Data

Initial velocity of train = v_i = 0 ms^{-1}

Distance covered by train = S = 1 km = 1000 m

Time taken by train = t = 100 s

Required

Speed of train after 100 s = v_f = ?

Solution

First we have to find the acceleration, as we know that

$S = v_i t + \frac{1}{2} at^2$

By putting the values, we have

$1000 = 0 \times 100 + \frac{1}{2} \times a \times (100)^2$

$1000 = \frac{1}{2} \times a \times 10000$

$1000 = a \times 5000$

$a = \frac{1000}{5000}$

So, $a = 0.2$ ms^{-2}

Now from first equation of motion, we have

$v_f = v_i + at$

by putting the values, we have

$v_f = 0 + 0.2 \times 100$

$v_f = 20\ ms^{-1}$

Result

Speed of train after 100 s = $v_f = 20\ ms^{-1}$

2.3 A car has a velocity of 10 ms^{-1}. It accelerates at 0.2 ms^{-2} for half minute. Find the distance traveled during this and the find velocity of the car.

Given Data

Velocity of the car = vi = 10 ms^{-1}

Acceleration of the car = a = 0.2 ms^{-2}

Time taken by car = t = 0.5 min. = 0.5 x 60 = 30 s

Required

Distance traveled by car = S = ?

Solution

As we know that

$$S = v_i t + \frac{1}{2} at^2$$

By putting the values, we have

$$S = 10 \times 30 + \frac{1}{2} \times 0.2 \times (30)^2$$

$$S = 300 + 0.1 \times 900$$

$$S = 300 + 90$$

$$S = 390\ m$$

Result

Distance traveled by car = S = 390 m

2.4 A tennis ball is hit vertically upward with a velocity of 30 ms^{-1}. It takes 3 s to reach the highest point. Calculate the maximum height reached by the ball. How long it will take to return to ground?

Given Data

Initial velocity of the tennis ball = vi = 30 ms^{-1}

Time to reach the maximum height = t = 3 s

Gravitational acceleration = g = -10 ms^{-2}

Final velocity of the ball = v_f = 0ms^{-1}

Required

Maximum height reached by the ball = h = ?

Solution

From second equation of motion in vertical motion, we have

$$h = v_i t + \frac{1}{2} gt^2$$

by putting the values, we have

$$h = 30 \times 3 + \frac{1}{2} \times (-10)\ (3)^2$$

$$h = 90 - 5 \times 9$$

$$h = 90 - 45$$

$$h = 45\ m$$

As the ball moves with uniform acceleration in vertical motion, so time taken by the ball in both directions will be same.

Total time taken to return the ground = Time taken upwards + Time taken downwards
Total time taken to return the ground = 3 s + 3s
Total time taken to return the ground = 6 s

Result

Maximum height reached by the ball = h = 45 m
Total time taken to return the ground = 6 s

2.5 A car moves with uniform velocity 40 ms^{-1} for 5 s. it comes to rest in the next 10 s with uniform declaration. Find
i) declaration
ii) total distance traveled by the car

Required

(i) Deceleration = $\vec{a}$ = ?
(ii) Distance traveled by the car = S = ?

Solution

(i) Slope of line BC = $\dfrac{y_2 - y_1}{x_2 - x_1}$

$$\vec{a} = \frac{0-40}{15-5} = \frac{-40}{10} = -4ms^{-2}$$

(ii) Area of trapezium OABC = $\frac{1}{2}$(sum of parallel sides)(perpendicular distance between parallel sides)

$$S = \frac{1}{2}(AB + OC)(OA)$$
$$S = \frac{1}{2}(5+15)(40)$$
$$= \frac{1}{2}(20)(40)$$
$$S = \frac{800}{2}$$
$$S = 400\,m$$

Result
Total distance moved by car = S = 400 m

2.6 A train start from rest with an acceleration of 0.5 ms^{-2}. Find its speed in kmh^{-1}, when it has moved through 100 m.

Given Data

Acceleration of the train = a = 0.5 ms^{-2}
Initial velocity of the train = v_i = 0 ms^{-1}
Distance moved by train = S = 100 m

Required

Final speed in kmh^{-1} = v_f = ?

Solution

From third equation of motion, we have
$$2aS = vf^2 - vi^2$$
by putting the values, we have
$$2 \times 0.5 \times 100 = vf^2 - (0)^2$$

$100 = vf^2$

by taking square root on both sides, we have

$$\sqrt{100} = v_f^{\,2}$$

So $\quad v_f = 10\ ms^{-1}$

In kmh-1

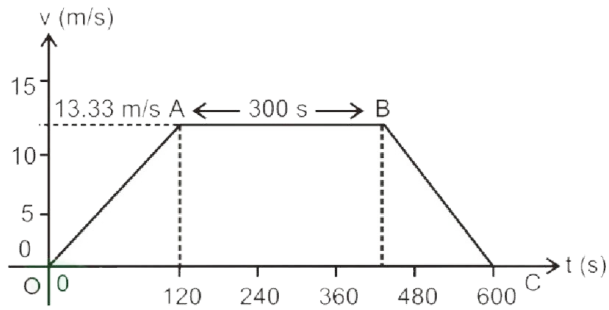
$$v_f = \frac{10 \times 3600}{1000}$$

$$v_f = 36\ kmh^{-1}$$

Result

Final speed in kmh^{-1} = v_f = 36 kmh^{-1}

2.7 A train starting from rest accelerates uniformly and attains a velocity 48 kmh^{-1} in 2 minutes. It travels at speed for 5 minutes. Finally, it moves with uniform retardation and is stopped after 3 minutes. Find the total distance traveled by the train.



Solution

Total distance covered=?

By using the given values we plot a graph shown in figure.

Velocity = 48 kmh^{-1}

$$= \frac{48 \times 1000}{3600}$$

$$= \frac{40}{3} ms^{-1}$$

time = 2 minutes

= 2(60)

= 120 S

Again time = 5 minutes

= 5(60)

= 300 S

Again time = 3 minutes

= 3(60)

= 180 S

We know that area under speed-time graph represents the distance covered by the object.

$\therefore$ Total distance covered = Area of trapezium OABC

$$= \frac{1}{2} \text{ (sum of parallel sides) (perpendicular distance between parallel sides)}$$

$$= \frac{1}{2}(600 + 300)\left(\frac{40}{3}\right)$$

$$= \frac{1}{2}(900)\left(\frac{40}{3}\right)$$

Total distance covered = 6000 m

Result

2.8 A cricket ball is hit vertically upwards and returns to ground 6 s later. Calculate
(i) Maximum height, reached by the ball.
(ii) Initial velocity of the ball.

Given Data

Final velocity of the ball = $v_f = 0\ ms^{-1}$
Gravitational acceleration = $g = -10\ ms^{-2}$
Time in which ball return to ground = $t = 6$ s

Required

Maximum height reached by ball = h = ?
Initial velocity of the ball = v_i = ?

Solution

We know that for ball thorwn vertically upward in air
Time taken by ball to reach maximum height = Time taken by ball to reach ground from maximum height
∴ time taken by ball to reach maximum height = t = 3 s
From first equation of motion, we have

$$v_f = v_i + gt$$

By putting the values, we have

$$0 = v_i + (-10) \times 3$$
$$0 = v_i - 30$$

So $v_i = 30\ ms^{-1}$

Now from second equation of motion, we have

$$S = v_i t + \tfrac{1}{2} gt^2$$

By putting the values, we have

$$S = 30 \times 3 + \tfrac{1}{2} \times (-10) \times (3)^2$$
$$S = 90 - 5 \times 9$$
$$S = 45\ m$$

Result

Maximum height reached by ball = h = 45 m
Initial velocity of the ball = $v_i = 30\ ms^{-1}$

2.9 When brakes are applied, the speed of a train decreases from 96 kmh^{-1} to 48 kmh^{-1} in 800 m. How much further will the train move before coming to rest? (Assuming the retardation to be constant)

Given Data

Initial velocity of train = $v_i = 96\ kmh^{-1} = \dfrac{96 \times 1000}{3600} = \dfrac{80}{3} ms^{-1}$

Final velocity of train = $v_f = 48\ kmh^{-1} = \dfrac{48 \times 1000}{3600} = \dfrac{40}{3} ms^{-1}$

Distance covered by train = 800 m

Required

Retardation of the train = a = ?

Solution

From third equation of motion, we have

$$2aS = v_f^2 - v_i^2$$

By putting the values, we have

$$2a\,(800) = \left(\frac{40}{3}\right)^2 - \left(\frac{80}{3}\right)^2$$

$$1600\,a = \frac{1600}{9} - \frac{6400}{9}$$

$$1600\,a = \frac{1600 - 6400}{9}$$

$$1600\,a = -\frac{4800}{9}$$

$$a = -\frac{4800}{9 \times 1600}$$

$$a = -\frac{1}{3}\,ms^{-2}$$

Again

Initial velocity of train = v_i = 48 $kmh^{-1} = \frac{40}{3} ms^{-1}$

Final velocity of train = v_f = 0 ms^{-1}

retardation of train = a = $-\frac{1}{3} ms^{-2}$

Required

Distance covered by train = S = ?

Solution

From third equation of motion, we have

$$2aS = v_f^2 - v_i^2$$

By putting the values, we have

$$2\left(-\frac{1}{3}\right)S = (0)^2 - \left(\frac{40}{3}\right)^2$$

$$-\frac{2}{3}S = -\frac{1600}{9}$$

$$S = \frac{1600}{9} \times \frac{3}{2}$$

$$S = 266.66\,m$$

Result

The train will move by 266.66 m before coming to rest

2.10 In the above problem, find the time taken by the train to stop after the application of the brakes.

Given Data

Initial velocity of train = v_i = 96 kmh^{-1} $= \frac{96 \times 1000}{3600} = \frac{80}{3} ms^{-1}$

Final velocity of train = v_f = 0 ms^{-1}

retardation of train = a = $-\frac{1}{3} ms^{-2}$

Required

Time taken by the train = t = ?

Solution

From first equation of motion, we have

$$v_f = v_i + at$$

By putting the values, we have

$$0 = \frac{80}{3} + \left(-\frac{1}{3}\right)t$$

$$0 = \frac{80}{3} - \frac{t}{3}$$

$$\frac{t}{3} = \frac{80}{3}$$

$$t = \frac{80}{3} \times 3$$

$$t = 80\text{s}$$

Result

Required time is 80 s.